SADLER MATHEMATICS METHODS UNIT 3

WORKED SOLUTIONS

Chapter 7 Calculus of trigonometric functions

Exercise 7A

Question 1

$$\frac{dy}{dx} = 5x^4 - 2x$$

Question 2

$$\frac{dy}{dx} = 3x^2$$

Question 3

$$\frac{dy}{dx} = \sin x$$

Question 4

$$\begin{aligned}\frac{dy}{dx} &= \cos x - (-\sin x)\\ &= \cos x + \sin x\end{aligned}$$

Question 5

$$\frac{dy}{dx} = -\sin x - \cos x$$

Question 6

$$\frac{dy}{dx} = 1 - \frac{1}{\cos^2 x}$$
$$= \frac{\cos^2 x - 1}{\cos^2 x}$$
$$= \frac{-\sin^2 x}{\cos^2 x}$$
$$= -\tan^2 x$$

Question 7

$$\frac{dy}{dx} = (x+1) \times 2 + (2x-3) \times 1$$
$$= 2x + 2 + 2x - 3$$
$$= 4x - 1$$

Question 8

$$\frac{dy}{dx} = 5x^2 \times (-5) + (1-5x) \times 10x$$
$$= -25x^2 + 10x - 50x^2$$
$$= -75x^2 + 10x$$

Question 9

$$\frac{dy}{dx} = 6\cos x$$

Question 10

$$\frac{dy}{dx} = 4(-\sin x)$$
$$= -4\sin x$$

Question 11

$$\frac{dy}{dx} = x(\cos x) + \sin x$$
$$= x\cos x + \sin x$$

Question 12

$$\begin{aligned}\frac{dy}{dx} &= x^2(-\sin x) + \cos x \times 2x \\ &= 2x\cos x - x^2 \sin x\end{aligned}$$

Question 13

$$\begin{aligned}\frac{dy}{dx} &= \frac{(3x^2-1)\times 1 - x(6x)}{(3x^2-1)^2} \\ &= \frac{3x^2-1-6x^2}{(3x^2-1)^2} \\ &= \frac{-3x^2-1}{(3x^2-1)^2} \\ &= -\frac{(3x^2+1)}{(3x^2-1)^2}\end{aligned}$$

Question 14

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x^2-1)\times 2x - (x^2+1)\times 2x}{(x^2-1)^2} \\ &= \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2} \\ &= -\frac{4x}{(x^2-1)^2}\end{aligned}$$

Question 15

$$\begin{aligned}\frac{dy}{dx} &= \frac{x(-\sin x) - \cos x \times 1}{x^2} \\ &= \frac{-x\sin x - \cos x}{x^2} \\ &= \frac{-(x\sin x + \cos x)}{x^2} \\ &= -\frac{x\sin x + \cos x}{x^2}\end{aligned}$$

Question 16

$$\frac{dy}{dx}=\frac{x(\cos x)-\sin x\times 1}{x^2}$$
$$=\frac{x\cos x-\sin x}{x^2}$$

Question 17

$$\frac{dy}{dx}=\frac{\sin x\times 1-x\cos x}{\sin^2 x}$$
$$=\frac{\sin x-x\cos x}{\sin^2 x}$$

Question 18

$$\frac{dy}{dx}=\frac{\cos x\times 1-x(-\sin x)}{\cos^2 x}$$
$$=\frac{\cos x+x\sin x}{\cos^2 x}$$

Question 19

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx} \qquad u=x^2+1$$
$$=6u\times 2x \qquad \frac{du}{dx}=2x$$
$$=12x(x^2+1)$$

Question 20

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx} \qquad u=x^2-1$$
$$=\frac{1}{2}u^{-\frac{1}{2}}\times 2x \qquad \frac{du}{dx}=2x$$
$$=\frac{x}{\sqrt{x^2-1}}$$

Question 21

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad u = 6x$$

$$= \cos u \times 6 \qquad \frac{du}{dx} = 6$$

$$= 6\cos 6x$$

Question 22

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad u = 2x + 3$$

$$= -\sin u \times 2 \qquad \frac{du}{dx} = 2$$

$$= -2\sin(2x+3)$$

Question 23

$$y = \sin^2 x = (\sin x)^2$$

$$\frac{dy}{dx} = 2\sin x \cos x$$

Question 24

$$\frac{dy}{dx} = 3\sin^2 x \cos x$$

Question 25

$$\frac{dy}{dx} = 5\cos^4 x(-\sin x)$$

$$= -5\cos^4 x \sin x$$

Question 26

$$\frac{dy}{dx} = -\sin 3x \times 3$$

$$= -3\sin 3x$$

Question 27

$$\frac{dy}{dx} = 3\cos(3x - 7)$$

Question 28

$$\begin{aligned}\frac{dy}{dx} &= 2(-\sin(2x + 5)) \\ &= -2\sin(2x + 5)\end{aligned}$$

Question 29

$$\begin{aligned}\frac{dy}{dx} &= -3(-\sin x) \\ &= 3\sin x\end{aligned}$$

Question 30

$$\begin{aligned}\frac{dy}{dx} &= 3 + 2(-\sin x) \\ &= 3 - 2\sin x\end{aligned}$$

Question 31

$$\frac{dy}{dx} = 2\cos 2x$$

Question 32

$$\begin{aligned}\frac{dy}{dx} &= 2x - (-\sin x) \\ &= 2x + \sin x\end{aligned}$$

Question 33

$$\frac{dy}{dx}=\frac{x^2(\cos x)-(1+\sin x)\times 2x}{x^4}$$
$$=\frac{x^2\cos x-2x(1+\sin x)}{x^4}$$
$$=\frac{x\cos x-2(1+\sin x)}{x^3}$$
$$=\frac{x\cos x-2\sin x-2}{x^3}$$

Question 34

$$\frac{dy}{dx}=3\cos x-2(-\sin x)$$
$$=3\cos x+2\sin x$$

Question 35

$$\frac{dy}{dx}=-\sin 3x\times 3$$
$$=-3\sin 3x$$

Question 36

$$\frac{dy}{dx}=-9\sin 9x$$

Question 37

$$\frac{dy}{dx}=3(-\sin 2x)\times 2$$
$$=-6\sin 2x$$

Question 38

$$\frac{dy}{dx}=5\cos 3x\times 3$$
$$=15\cos 3x$$

Question 39

$$\frac{dy}{dx} = 2\cos 3x \times 3 + 3(-\sin 2x) \times 2$$
$$= 6\cos 3x - 6\sin 2x$$

Question 40

$$\frac{dy}{dx} = 5\sin^4 x \cos x$$

Question 41

$$\frac{dy}{dx} = 5 \times 2\cos x(-\sin x)$$
$$= -10\cos x \sin x$$

Question 42

$$\frac{dy}{dx} = \frac{1}{2}(\sin x)^{-\frac{1}{2}}\cos x$$
$$= \frac{\cos x}{2\sqrt{\sin x}}$$

Question 43

$$f'(x) = 7\cos 7x$$

Question 44

$$f'(x) = 8\cos 8x$$

Question 45

$$f'(x) = \cos 4x \times 4 + (-\sin 4x) \times 4$$
$$= 4\cos 4x - 4\sin 4x$$

Question 46

$$\begin{aligned} f'(x) &= 2 \times \cos(3x-1) \times 3 \\ &= 6\cos(3x-1) \end{aligned}$$

Question 47

$$\begin{aligned} f'(x) &= 4\left[-\sin(4x+3)\right] \times 4 \\ &= -16\sin(4x+3) \end{aligned}$$

Question 48

$$\begin{aligned} f'(x) &= 2 \times 3\sin^2 x \times \cos x \\ &= 6\sin^2 x\cos x \end{aligned}$$

Question 49

$$\begin{aligned} f'(x) &= 3 \times 2\cos x(-\sin x) \\ &= -6\cos x\sin x \end{aligned}$$

Question 50

$$\begin{aligned} f'(x) &= x(-\sin x) + \cos x \times 1 \\ &= \cos x - x\sin x \end{aligned}$$

Question 51

$$\begin{aligned} f'(x) &= x^2(-\sin x) + \cos x \times 2x \\ &= 2x\cos x - x^2\sin x \end{aligned}$$

Question 52

$$\begin{aligned} f'(x) &= 2x \times \cos x + \sin x \times 2 \\ &= 2x\cos x + 2\sin x \end{aligned}$$

Question 53

$$f'(x) = 2\left[\frac{\cos x \times \cos x - \sin x(-\sin x)}{\cos^2 x}\right]$$
$$= 2\frac{(\cos^2 x + \sin^2 x)}{\cos^2 x}$$
$$= \frac{2}{\cos^2 x}$$

Question 54

$$f(x) = 2\tan x$$
$$f'(x) = \frac{2}{\cos^2 x}$$

Question 55

$$\frac{dy}{dx} = \cos x$$

At $x = \frac{\pi}{6}$

$$\frac{dy}{dx} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Question 56

$$\frac{dy}{dx} = -2\sin 2x$$

At $x = \frac{\pi}{6}$,

$$\frac{dy}{dx} = -2\sin\left(2 \times \frac{\pi}{6}\right)$$
$$= -2\sin\left(\frac{\pi}{3}\right)$$
$$= -2 \times \frac{\sqrt{3}}{2}$$
$$= -\sqrt{3}$$

Question 57

$$\frac{dy}{dx} = 2(\sin x(-\sin x) + \cos x \times \cos x)$$
$$= 2(\cos^2 x - \sin^2 x)$$

At $x = 0$,

$$\frac{dy}{dx} = 2(\cos^2 0 - \sin^2 0)$$
$$= 2(1-0)$$
$$= 2$$

Question 58

$$\frac{dy}{dx} = 3\sin x \times \cos x$$
$$= 6\sin x \cos x$$

At $x = \pi$,

$$\frac{dy}{dx} = 6\sin \pi \cos \pi$$
$$= 0$$

Question 59

$$\frac{dy}{dx} = \cos x$$
$$\frac{d^2 y}{dx^2} = -\sin x$$

Question 60

$$\frac{dy}{dx} = -5\sin 5x$$
$$\frac{d^2 y}{dx^2} = -5 \times 5\cos 5x$$
$$= -25\cos 5x$$

Question 61

$$\frac{dy}{dx} = 3\cos 2x \times 2$$
$$= 6\cos 2x$$
$$\frac{d^2y}{dx^2} = 6(-\sin 2x) \times 2$$
$$= -12\sin 2x$$

Question 62

$$\frac{dy}{dx} = \cos x - \sin x$$
$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

Question 63

$$\frac{dy}{dx} = x\cos x + \sin x \times 1$$
$$= x\cos x + \sin x$$

At $x = \frac{\pi}{2}$,

$$\frac{dy}{dx} = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$$
$$= 0 + 1$$
$$= 1$$

Equation of tangent

$y = 1x + c$

Using $(\frac{\pi}{2}, \frac{\pi}{2})$

$$\frac{\pi}{2} = \frac{\pi}{2} + c$$
$$c = 0$$
$$\therefore y = x$$

Question 64

$$\frac{dy}{dx} = 1 + 3(-\sin 2x) \times 2$$
$$= 1 - 6\sin 2x$$

At $x = 0$,

$$\frac{dy}{dx} = 1 - 6\sin 0$$
$$= 1$$

Equation of tangent

$y = x + c$

Using (0,3)

$3 = 0 + c$

$c = 3$

$\therefore y = x + 3$

Question 65

a $f'(x) = 2\cos 2x$

$$f'\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{3}$$
$$= 1$$

b $f''(x) = -4\sin 2x$

$$f''\left(\frac{\pi}{6}\right) = -4\sin 2 \times \frac{\pi}{3}$$
$$= -4 \times \frac{\sqrt{3}}{2}$$
$$= -2\sqrt{3}$$

Question 66

$$y = \sin x^\circ = \sin\left(\frac{\pi x}{180}\right)$$

$$\frac{dy}{dx} = \frac{\pi}{180} \times \cos\left(\frac{\pi x}{180}\right)$$
$$= \frac{\pi}{180}\cos x^\circ$$

Question 67

$$A = 2h \times 2w$$
$$= 4hw$$

$$\sin\theta = \frac{h}{10} \Rightarrow h = 10\sin\theta$$

$$\cos\theta = \frac{w}{10} \Rightarrow w = 10\cos\theta$$

$$A = 4 \times 10\sin\theta \times 10\cos\theta$$
$$= 400\sin\theta\cos\theta$$

$$\frac{dA}{d\theta} = 400(\sin\theta(-\sin\theta) + \cos\theta\cos\theta)$$
$$= 400(\cos^2\theta - \sin^2\theta)$$
$$= 400(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$$

$$400(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 0$$

$$\cos\theta + \sin\theta = 0 \text{ or } \cos\theta - \sin\theta = 0$$

$$\cos = -\sin\theta \text{ or } \cos\theta = \sin\theta$$

$$\tan\theta = -1 \text{ or } \tan\theta = 1$$

Given $0 < \pi < \frac{\pi}{2}, \theta = \frac{\pi}{4}$

$$h = 10\sin\frac{\pi}{4}$$
$$= 5\sqrt{2}$$

$$w = 10\cos\frac{\pi}{4}$$
$$= 5\sqrt{2}$$

Dimensions of rectangle

$$2h \times 2w = 10\sqrt{2} \times 10\sqrt{2}$$

Hence the rectangle is a square

$$A = 4hw$$
$$= 4 \times 5\sqrt{2} \times 5\sqrt{2}$$
$$= 200 \text{ cm}^2$$

Question 68

$$\text{Area} = \frac{1}{2}\times 10\times 8\times \sin 0.1t$$
$$= 40\sin 0.1t \text{ cm}^2$$
$$\frac{dA}{dt} = 40\cos(0.1t)\times 0.1$$
$$= 4\cos 0.1t \text{ cm}^2/\text{s}$$

a When $t = 1$,

$$\frac{dA}{dt} = 4\cos 0.1$$
$$= 3.98 \text{ cm}^2/\text{s}$$

b When $t = 5$,

$$\frac{dA}{dt} = 4\cos 0.5$$
$$= 3.51 \text{ cm}^2/\text{s}$$

c When $t = 10$,

$$\frac{dA}{dt} = 4\cos 1$$
$$= 2.16 \text{ cm}^2/\text{s}$$

d When $t = 20$,

$$\frac{dA}{dt} = 4\cos 2$$
$$= -1.66 \text{ cm}^2/\text{s}$$

Question 69

a $x = 5\sin 3t, t \geq 0$

Max value is 5 as the maximum value of $\sin 3t = 1$

$5 = 5\sin 3t$

$\sin 3t = 1$

$$3t = \frac{\pi}{2}$$

$$t = \frac{\pi}{6}$$

b $5\sin 3t = 2.5$

$\sin 3t = 0.5$

$$3t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$t = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}$$

c $\frac{dx}{dt} = 5\cos 3t \times 3$

$= 15\cos 3t$

At $t = 0.6$,

$\frac{dx}{dt} = 15 \times \cos 1.8$

$= -3.4$

d $\frac{d^2x}{dt^2} = -15\sin 3t \times 3$

$= -45\sin 3t$

$= -9 \times 5\sin 3t$

$\therefore k = -9$

Question 70

$$\frac{dy}{dx} = 3\cos\theta + 4(-\sin\theta)$$

$$0 = 3\cos\theta - 4\sin\theta$$

$$4\sin\theta = 3\cos\theta$$

$$\frac{4\sin\theta}{4\cos\theta} = \frac{3\cos\theta}{4\cos\theta}$$

$$\tan\theta = \frac{3}{4}$$

$$\theta = 0.6435$$

Maximum value

$$3\sin(0.6435) + 4\cos(0.6435) = 5$$

Exercise 7B

Question 1

$$5\int \cos x \ dx = 5\sin x + c$$

Question 2

$$2\int \sin x \ dx = -2\cos x + c$$

Question 3

$$\begin{aligned} -10\int \sin x \ dx &= -10(-\cos x) + c \\ &= 10\cos x + c \end{aligned}$$

Question 4

$$-2\int \cos x \ dx = -2\sin x + c$$

Question 5

$$3\int 2\cos 2x \ dx = 3\sin 2x + c$$

Question 6

$$\frac{1}{3}\int 6\cos 6x \ dx = \frac{1}{3}\sin 6x + c$$

Question 7

$$3\int 4\sin 4x \ dx = -3\cos 4x + c$$

Question 8

$$-\frac{1}{3}\int 3\sin 3x \ dx = -\frac{1}{3}(-\cos 3x) + c$$
$$= \frac{1}{3}\cos 3x + c$$

Question 9

$$-\frac{8}{10}\int (10\cos 10x) \ dx = -\frac{4}{5}\sin 10x + c$$

Question 10

$$2\int \frac{1}{2}\sin\frac{x}{2} \ dx = 2\left(-\cos\frac{x}{2}\right) + c$$
$$= -2\cos\frac{x}{2} + c$$

Question 11

$$\frac{2}{3}\int \frac{3}{2}\cos\frac{3x}{2} \ dx = \frac{2}{3}\sin\frac{3x}{2} + c$$

Question 12

$$-9\int \frac{2}{3}\sin\frac{2x}{3} \ dx = -9(-\cos\frac{2x}{3}) + c$$
$$= 9\cos\frac{2x}{3} + c$$

Question 13

$$3\int 2\sin(2x+3) \ dx = -3\cos(2x+3) + c$$

Question 14

$$\frac{3}{2}\int 2\cos(2x-3) \ dx = \frac{3}{2}\sin(2x-3) + c$$

Question 15

$$\frac{1}{2}\int 2\cos\left(2x+\frac{2\pi}{3}\right)\,dx=\frac{1}{2}\sin\left(2x+\frac{2\pi}{3}\right)+c$$

Question 16

$$\begin{aligned}-1\int\left(-\sin(-x)\right)\,dx&=-\left(-\cos(-x)\right)+c\\&=\cos(-x)+c\\&=\cos x+c\end{aligned}$$

Question 17

$$4\int\frac{1}{\cos^2 x}\,dx=4\tan x+c$$

Question 18

$$3\int 2\cos 2x\,dx+2\int 3\sin 3x\,dx=3\sin 2x-2\cos 3x+c$$

Question 19

$$\begin{aligned}\int(\cos 8x-4\sin 2x)\,dx&=\frac{1}{8}\int 8\cos 8x\,dx-2\int 2\sin 2x\,dx\\&=\frac{1}{8}\sin 8x+2\cos 2x+c\end{aligned}$$

Question 20

$$\int(2x+4\cos x+6\cos 2x)\,dx=x^2+4\sin x+3\sin 2x+c$$

Question 21

$$\int(3+4x-6x^2+10\cos 5x-2\sin 4x)\,dx=3x+2x^2-2x^3-2\sin 5x+\frac{1}{2}\cos 4x+c$$

Question 22

$$\int\cos^3 x\sin x\,dx=-\frac{1}{4}\cos^4 x+c$$

Question 23

$$\begin{aligned}-30\int \cos^5 x(-\sin x)\,dx &= -30\frac{\cos^6 x}{6}+c\\ &= -5\cos^6 x + c\end{aligned}$$

Question 24

$$\begin{aligned}\int (\sin 5x\cos 2x+\cos 5x\sin 2x)\ dx &= \int \sin 7x dx\\ &= -\frac{1}{7}\cos x + c\end{aligned}$$

Question 25

$$\begin{aligned}\int (\sin 3x\cos x-\cos 3x\sin x)\ dx &= \int \sin 2x dx\\ &= -\frac{1}{2}\cos 2x + c\end{aligned}$$

Question 26

$$\begin{aligned}\int (\cos 5x\cos 2x-\sin 5x\sin 2x)\ dx &= \int \cos 7x dx\\ &= \frac{1}{7}\sin 7x + c\end{aligned}$$

Question 27

$$\begin{aligned}\int (\cos 5x\cos x+\sin 5x\sin x)\ dx &= \int \cos 4x dx\\ &= \frac{1}{4}\sin 4x + c\end{aligned}$$

Question 28

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin x\ dx &= \left[-\cos x\right]_0^{\frac{\pi}{2}}\\ &= -\cos\frac{\pi}{2}-(-\cos 0)\\ &= 0+1\\ &= 1\end{aligned}$$

Question 29

$$\int_0^{\frac{\pi}{2}} \cos x \ dx = [\sin x]_0^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin 0$$

$$= 1$$

Question 30

$$\int_{\frac{\pi}{2}}^{\pi} \cos\frac{x}{2} \ dx = \left[2\sin\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= 2\sin\frac{\pi}{2} - 2\sin\frac{\pi}{4}$$

$$= 2 - \sqrt{2}$$

Question 31

a $\int_0^{\frac{\pi}{4}} \sin x \ dx$

$$= [-\cos x]_0^{\frac{\pi}{4}}$$

$$= -\cos\left(\frac{\pi}{4}\right) - (-\cos 0)$$

$$= -\frac{1}{\sqrt{2}} + 1$$

$$= 1 - \frac{1}{\sqrt{2}}$$

b $\int_{\frac{\pi}{4}}^{0} \sin x \ dx$

$$= [-\cos x]_0^{\frac{\pi}{4}}$$

$$= -\cos 0 - \left[-\cos\left(\frac{\pi}{4}\right)\right]$$

$$= -1 + \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} - 1$$

Question 32

$$\int_0^{\pi} \sin x \; dx = \left[-\cos x\right]_0^{\pi}$$
$$= -\cos \pi - (-\cos 0)$$
$$= -(-1) - (-1)$$
$$= 1 + 1$$
$$= 2 \text{ units}^2$$

Question 33

a $$\int_{\pi}^{\frac{4\pi}{3}} \sin x \; dx$$
$$= \left[-\cos x\right]_{\pi}^{\frac{4\pi}{3}}$$
$$= -\cos \frac{4\pi}{3} - (-\cos \pi)$$
$$= -\left(-\frac{1}{2}\right) - 1$$
$$= \frac{1}{2} - 1$$
$$= -\frac{1}{2}$$
$$\therefore \text{Area} = 0.5 \text{ units}^2$$

b $$\int_{\pi}^{\frac{4\pi}{3}} \sin x \; dx + \int_0^{\pi} \sin x \; dx$$
$$= 2 + \frac{1}{2}$$
$$= 2.5$$
$$\therefore \text{Area} = 2.5 \text{ units}^2$$

Question 34

a $\frac{dv}{dt} = -2\sin 2t \times 2 = 0$

$$\sin 2t = 0$$
$$2t = 0,\ \pi$$
$$t = 0,\ \frac{\pi}{2}$$

$v = 2\cos 0$

$= 2$ m/s

b $\int 2\cos 2t\ dt$

$= \sin 2t + c$

When $t = 0,\ x = 5$

$x = \sin 2t + c$

$5 = \sin 2t + c$

$5 = \sin 0 + c$

$c = 5$

$\therefore x = (\sin 2t + 5)$ metres

c $x = 5 + \sin 2t$

Minimum value $\sin 2t$ is -1.

$\therefore$ Minimum distance is 4 metres.

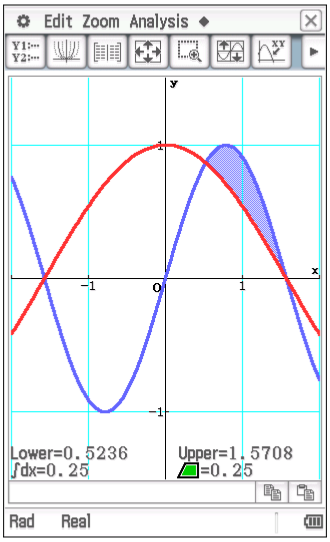
d $a = \frac{dv}{dt}$

$= -2\sin 2t \times 2$

$= -4\sin 2t$ m/s^2

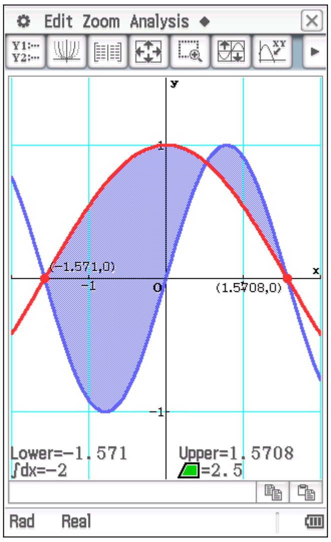
Question 35

a



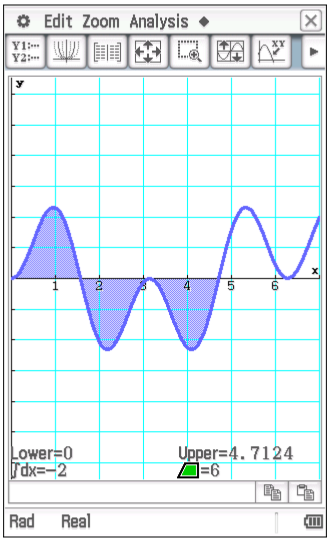
Area: 0.25 square units

b



Area: 2.5 square units

Question 36



a $y = 6\cos x \sin^2 x = 0$

$\cos x = 0$ or $\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad x = 0, \pi, 2\pi$

$A(\frac{\pi}{2}, 0)$, $B(\pi, 0)$, $C(\frac{3\pi}{2}, 0)$

b Area of 6 square units.

Miscellaneous exercise seven

Question 1

$$\frac{dy}{dx} = e^x$$

Question 2

$$\frac{dy}{dx} = 2e^x$$

Question 3

$$\frac{dy}{dx} = 8e^x$$

Question 4

$$\frac{dy}{dx} = e^x + \cos x$$

Question 5

$$\begin{aligned}\frac{dy}{dx} &= e^{\cos x}(-\sin x)\\ &= -\sin x e^{\cos x}\end{aligned}$$

Question 6

$$\begin{aligned}\frac{dy}{dx} &= e^{\sin 2x} \times \cos 2x \times 2\\ &= 2\cos 2x e^{\sin 2x}\end{aligned}$$

Question 7

$$\begin{aligned}\frac{dy}{dx} &= e^{2\sin x} \times 2\cos x\\ &= 2\cos x e^{2\sin x}\end{aligned}$$

Question 8

$$\begin{aligned}\frac{dy}{dx}&=e^x-x^{-2}\\&=e^x-\frac{1}{x^2}\end{aligned}$$

Question 9

$$\begin{aligned}\frac{dy}{dx}&=4\times\frac{1}{2}x^{-\frac{1}{2}}+e^{3x}\times 3\\&=\frac{2}{\sqrt{x}}+3e^{3x}\end{aligned}$$

Question 10

$$\begin{aligned}\frac{dy}{dx}&=e^x\times\frac{1}{2}x^{-\frac{1}{2}}+\sqrt{x}\times e^x\\&=\frac{e^x}{2\sqrt{x}}+\sqrt{x}e^x\\&=\frac{e^x+2\sqrt{x}\times\sqrt{x}e^x}{2\sqrt{x}}\\&=\frac{e^x+2\times x\times e^x}{2\sqrt{x}}\\&=\frac{e^x(2x+1)}{2\sqrt{x}}\end{aligned}$$

Question 11

$$\begin{aligned}\frac{dy}{dx}&=e^x\times\cos x+\sin x\times e^x\\&=e^x(\cos x+\sin x)\end{aligned}$$

Question 12

$$\begin{aligned}\frac{dy}{dx}&=e^x\times(-\sin 2x\times 2)+\cos 2x\times e^x\\&=e^x(\cos 2x-2\sin 2x)\end{aligned}$$

Question 13

$$\frac{dy}{dx} = e^x \times 2\sin x\cos x + \sin^2 xe^x$$
$$= e^x \sin x(2\cos x + \sin x)$$

Question 14

$$\frac{dy}{dx} = e^{3x^2+2} \times (6x)$$
$$= 6xe^{3x^2+2}$$

Question 15

$$\frac{dy}{dx} = (2x + \cos x)e^{x^2+\sin x}$$

Question 16

$$\frac{dT}{dr} = 3(2r+3)^2 \times 2$$
$$= 6(2r+3)^2$$

Question 17

a $\int_0^2 4e^{2x}dx$

$= 2\int_0^2 2 \times e^{2x}dx$

$= 2\left[e^{2x}\right]_0^2$

$= 2\left[e^4 - e^0\right]$

$= 2(e^4 - 1)$

b $\int_2^5 x^{-2}dx$

$= \left[\frac{x^{-1}}{-1}\right]_2^5$

$= \left[-\frac{1}{x}\right]_2^5$

$= -\frac{1}{5} - \left(-\frac{1}{2}\right)$

$= \frac{3}{10}$

c $\int_1^2 30(2x-3)^4 dx$

$= 15\int_1^2 2(2x-3)^4 dx$

$= 15\left[\frac{(2x-3)^5}{5}\right]_1^2$

$= 15\left[\frac{(4-3)^5}{5} - \frac{(2-3)^5}{5}\right]$

$= 15\left[\frac{1}{5} + \frac{1}{5}\right]$

$= 6$

Question 18

$$\frac{dy}{dx} = e^x \times \cos x + \sin x \times e^x$$

$$0 = e^x(\cos x + \sin x)$$

$$\cos x + \sin x = 0 \qquad (e^x \neq 0)$$

$$\cos x = -\sin x$$

$$\tan x = -1$$

$$x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

Question 19

$$\frac{dy}{dx} = e^{-x} \times \cos x + \sin x \times e^{-x} \times (-1)$$

$$= e^{-x}\cos x - \sin x \times e^{-x}$$

$$= e^{-x}(\cos x - \sin x)$$

When $x = \pi$,

$$\frac{dy}{dx} = e^{-\pi}(\cos \pi - \sin \pi)$$

$$= e^{-\pi}(-1 - 0)$$

$$= -e^{-\pi}$$

Question 20

$\lim_{h \to 0}\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)$ is the derivative of $y = \sqrt{x}$ via first principles.

$\therefore 0.5x^{-0.5}$

Question 21

$$A = \frac{3\ 000\ 000}{e^{0.1t}}$$

As t increases, $e^{0.1t}$ increases. $\therefore$ A is decreasing.

a

$$\frac{dA}{dt} = 3\ 000\ 000e^{-0.1t} \times (-0.1)$$

$$= -300\ 000e^{-0.1t}$$

When $t = 2$,

$$\frac{dA}{dt} = 300\ 000e^{-0.1\times 2}$$

$$= -245\ 619.23$$

$\therefore$ $-245\ \ 619$ tonnes per year (nearest 1 tonne/year)

b

When $t = 5$,

$$\frac{dA}{dt} = 300\ 000e^{-0.1\times 5}$$

$$= -181\ 959.20$$

$\therefore$ $-181\ 959$ tonnes per year (nearest 1 tonne/year)

c

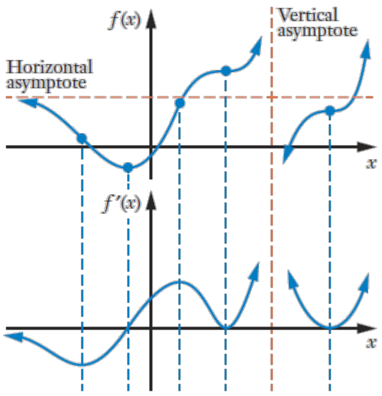
When $t = 10$,

$$\frac{dA}{dt} = 300000e^{-0.1\times 10}$$

$$= -110363.83$$

$\therefore$ -110364 tonnes per year (nearest 1 tonne/year)

Question 22



Question 23

a 10 m/s

b acceleration is the derivative (gradient) of velocity

At t = 13, the gradient is $\frac{-10}{4} = -2.5$ m/s^2

c Area under graph from $t = 0$ to $t = 28$,

$$
\begin{array}{lll}
0 \le t < 5 & \frac{1}{2}\times 5\times 10 & = 25 \\
5 \le t < 12 & 7\times 10 & = 70 \\
12 \le t < 16 & \frac{1}{2}\times 10\times 4 & = 20 \\
16 \le t < 28 & \frac{1}{2}\times 12\times 5 & = 30
\end{array}
$$

$\therefore$ Distance $= 145$ m

d $25 + 70 + 20 - 30 = 85$ m

e $\mathrm{A} \to \mathrm{B} \to \mathrm{C}$

$\quad\quad \mathrm{B} \leftarrow$

Velocity is negative when $t > 16$, therefore particle is at C when $t = 16$.

$\therefore$ C is $25 + 70 + 20 = 115$ m from A.

f Particle rests ($v = 0$) at B when $t = 32$.

$\frac{1}{2}\times 16\times 5 = 40$

$\therefore$ B is 40 m from C.

$\therefore$ B is 75 m from A.

Question 24

a $V = \frac{4}{3}\pi(100-3x)^3 \ ,V \geq 0$

b $0.5 \times \frac{4}{3}\pi 100^3 = \frac{4}{3}\pi(100-3x)^3$

$$0.5 \times 100^3 = (100-3x)^3$$

$$100-3x = \sqrt[3]{500\,000}$$

$$= 79.37$$

$$3x = 20.63$$

$$x = 6.88$$

c $V = \frac{4}{3}\pi(100-3x)^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3(100-3x)^2 \times (-3)$$

$$= -12\pi(100-3x)^2$$

$\therefore$ Volume is decreasing by $12\pi(100-3x)^2$ m^3/day.

d When $x = 5$,

$$\frac{dV}{dt} = 12\pi(100-3\times 5)^2$$

$$= 12\pi \times 85^2$$

$= \sim 270\ 000$ m^3/day

e $\frac{dV}{dx} = \frac{4}{3}\pi \times 3(100-2x-x^2)^2 \times (-2-2x)$

$$= -4\pi(2+2x)(100-2x-x^2)^2$$

$$= -8\pi(1+x)(100-2x-x^2)^2$$

$\therefore$ Rate of loss : $8\pi(1+x)(100-2x-x^2)^2$

When $x = 5$,

$$\frac{dV}{dx} = 8\pi(6) \times (65)^2$$

$$= 637\ 114.99$$

$\therefore \sim 640\ 000$ m^3/day

Question 25

a When $t = 2$,

$V = 2(1 - e^{-0.2\times 2})$ m/s

$= 0.6594$

$\therefore$ Speed $= 0.66$ m/s

b $V = 2 - 2e^{-0.2t}$

$a = \dfrac{dV}{dt} = -0.2(-2e^{-0.2t})$

$= 0.4e^{-0.2t}$

When $t = 2$,

$a = 0.4e^{-0.2\times 2}$

$= 0.27 \text{ m/s}^2$

c When $t = 10$,

$a = 0.4e^{-0.2\times 10}$

$= 0.05 \text{ m/s}^2$

Question 26

a $\displaystyle\int_0^{\frac{5\pi}{6}} \left(\sin x - \frac{3x}{5\pi}\right) dx$

$$= \left[-\cos x - \frac{3}{5\pi} \times \frac{x^2}{2}\right]_0^{\frac{5\pi}{6}}$$

$$= \left[-\cos x - \frac{3x^2}{10\pi}\right]_0^{\frac{5\pi}{6}}$$

$$= \left(-\cos\frac{5\pi}{6} - \frac{3}{10\pi} \times \left(\frac{5\pi}{6}\right)^2\right) - (-\cos 0 - 0)$$

$$= \left(\frac{\sqrt{3}}{3} - \frac{5\pi}{24}\right) - (-1)$$

$$= \left(1 + \frac{\sqrt{3}}{2} - \frac{5\pi}{24}\right) \text{ units}^2$$

b $\dfrac{24 + 12\sqrt{3} - 5\pi}{12}$ units^2

$$= \left(2 + \sqrt{3} - \frac{5\pi}{12}\right) \text{ units}^2$$

